



# Classicality of the primordial perturbations

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## Abstract

We show that during inflation, a quantum fluctuation becomes classical at all orders if it becomes classical at first order. Implications are discussed.

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## 1. Introduction

The promotion of cosmology from an area of speculation to an area of observation and measurement is one of the most remarkable developments in the history of human knowledge. Starting at an ‘initial’ temperature of a few MeV, the observable Universe is understood in considerable detail (we are setting  $\hbar = c = k_B = 1$ ). At the initial epoch the expanding Universe is an almost isotropic and homogeneous gas. The perturbations away from perfect isotropy and homogeneity present at the initial epoch are the subject of intense study at present, because they determine the subsequent evolution of all future perturbations.

According to observation, the dominant initial perturbation is the curvature perturbation  $\zeta$ , so-called because it is related to the perturbation in the intrinsic curvature of spacetime slices with uniform energy density. Other initial perturbations may be detected in the future. The usually-considered examples are a perturbation in the composition of the cosmic fluid (an isocurvature perturbation), a tensor perturbation setting the initial amplitude of primordial gravitational waves, and a primordial magnetic field.

To understand the nature and origin of the initial perturbations, one should use comoving coordinates  $\mathbf{x}$  that move with expansion of the unperturbed Universe. After a mapping from

the unperturbed Universe to the perturbed one has been chosen, the perturbations are functions of these coordinates. Such a mapping is called a gauge, and indeed is analogous to the field-theoretic gauge, which also needs to be fixed if gauge fields play a role.

It is convenient to consider the Fourier components with comoving wave-vector  $\mathbf{k}$ . Physical positions are  $a(t)\mathbf{x}$  and physical wave-vectors are  $\mathbf{k}/a(t)$ , where  $a$  is the scale factor of the Universe, and one also defines the Hubble parameter  $H = \dot{a}/a$ .

The crucial point now is that gravity slows down the expansion of the gaseous cosmic fluid, which means that  $aH/k \equiv \dot{a}/k$  increases as we go back in time. At the present epoch scales of cosmological interest correspond to  $10^{-6} \lesssim aH/k \lesssim 1$ , but at the ‘initial’ temperature  $T \sim \text{MeV}$  they all correspond to  $aH/k \gg 1$ . Such scales are out of effective causal contact, because the rate of change of the inverse wavenumber,  $\dot{a}/k$ , is bigger than 1. They are said to be *outside the horizon*.

To explain the origin of the perturbations, it is supposed that going further back in time we reach an era of inflation,<sup>1</sup> when by definition  $aH/k$  decreases again to cross the horizon at the epoch  $aH = k$ . With mild assumptions, it can be shown that inflation drives all perturbations to zero at the classical level [1]. But each bosonic field has a vacuum fluctuation, and one

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<sup>1</sup> The term inflation usually denotes an era of expansion corresponding to  $\ddot{a} > 0$ , sometimes described as ‘repulsive gravity’. For the present purpose it might instead be one of contraction, corresponding to  $\ddot{a} < 0$ , though predictions in that scenario depend on the unknown physics of the bounce.

or more of these fluctuations is supposed to become classical around the time of horizon exit, the idea being that the timescale of the would-be fluctuation becomes longer than the Hubble time  $H^{-1}$ . On this basis, the correlators of the classical perturbation are identified with quantum expectation values which can be calculated once one has adopted a specific theory for the inflationary era. Finally, the classical evolution after horizon exit is supposed to produce the perturbations at  $T \sim \text{MeV}$ .

The nature of the evolution after horizon entry is not the concern of the present Letter. Rather, we want to show that the quantum-to-classical transition can actually occur. This issue has so far been addressed only in the context of first-order calculations, even though higher-order calculations [2–9] have now been done. We begin by reviewing the first-order case, keeping the discussion very general so as to include the quite wide variety of scenarios that have been proposed.

## 2. First order

Let  $\phi_{\mathbf{k}}$  be a Fourier component of any perturbation existing around the time of horizon exit. (It might be a scalar field, or else a component of a higher-spin bosonic field or of a metric perturbation.) To first order it will satisfy a linear evolution equation, which we take to be of the form

$$\phi_{\mathbf{k}}'' + \omega_{\mathbf{k}}^2(k, \eta)\phi_{\mathbf{k}} = 0, \quad (1)$$

where a prime denotes  $d/d\eta = a(t) d/dt$ . Well before horizon exit the expansion rate is negligible compared with the physical wavenumber  $k/a$ , so that we are dealing with flat spacetime and a slowly-varying angular frequency of the form  $\omega_{\mathbf{k}} = c_s(k, \eta)k$ . Usually  $c_s = 1$  corresponding to a canonically normalized field with negligible mass, but scenarios [3,10] exist with  $c_s \ll 1$ . In the latter case the epoch of horizon exit (when the quantum-to-classical transition occurs) should be redefined as  $c_s k \sim aH$ .

In writing Eq. (1) we suppose that the perturbations can be chosen so that there are no linear couplings between them, which would make the right hand side a linear combination of the other perturbations. This will usually be a good approximation as we deal only with the few Hubble times around horizon exit. The inclusion of linear couplings makes no difference, provided that they are negligible well before horizon exit, as they certainly are in the usual case corresponding to  $c_s = 1$ .

In the Heisenberg picture the corresponding operator  $\hat{\phi}_{\mathbf{k}}$  has the classical time-dependence, and the reality condition  $\hat{\phi}_{-\mathbf{k}} = \hat{\phi}_{\mathbf{k}}^\dagger$  determines its form as  $\hat{\phi}_{\mathbf{k}}(\eta) = v_{\mathbf{k}}^*(\eta)\hat{a}_{\mathbf{k}} + v_{\mathbf{k}}(\eta)\hat{a}_{-\mathbf{k}}^\dagger$ , where the mode function  $v_{\mathbf{k}}(\eta)$  also satisfies Eq. (1). To determine the quantum theory we need only consider the epoch well before horizon exit. Starting from the action for  $\phi_{\mathbf{k}}$  we apply canonical quantization, which after fixing the normalization of the commutator  $[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] \propto \delta(\mathbf{k} - \mathbf{k}')$  determines the Wronskian  $v_{\mathbf{k}}' v_{\mathbf{k}}^* - v_{\mathbf{k}} v_{\mathbf{k}}'^*$  of the mode function. Then we choose a particular mode function, which is usually taken to be [11,12] be  $v_{\mathbf{k}} \propto e^{\pm i c_s k \eta}$ . (The  $\pm$  choices are physically equivalent; a mixture  $v_{\mathbf{k}} = \alpha_k e^{+i c_s k \eta} + \beta_k e^{-i c_s k \eta}$  is sometimes considered [13].) Next we use the creation operator  $\hat{a}_{\mathbf{k}}^\dagger$  to construct the Hilbert space (Fock space), starting from the vacuum annihilated by  $\hat{a}_{\mathbf{k}}$ . A similar Hilbert space is constructed for every component of

every field. The complete Hilbert space is the direct product of these, together with another Hilbert space describing the fields which do not correspond to perturbations and are irrelevant in the free-field approximation. Finally, the state vector is specified, which describes our Universe well before horizon entry. This setup defines, at any instant in time, an ensemble of universes specified by different values of  $\phi_{\mathbf{k}}$ . The ensemble corresponds to a Gaussian random field, meaning that the two-point correlator  $\langle \hat{\phi}_{\mathbf{k}} \hat{\phi}_{\mathbf{k}'} \rangle$  is the only connected one.

Now comes the point. We make the usual assumption that the state vector corresponds to the vacuum, for at least [14,15] most of the states.<sup>2</sup> Then the fluctuation  $\phi_{\mathbf{k}}$  well before horizon entry is definitely a quantum object; the wave function giving its probability distribution is so narrowly peaked that no wave-packet state vectors exist, such that  $\phi_{\mathbf{k}}(t)$  has a well defined value. (We are talking about values with reasonable probability of course, as opposed to large values far out on the tail of the probability distribution. Our universe is supposed to correspond to a typical member of the ensemble, and therefore the perturbations we measure are not supposed to deviate too much from the ensemble average.)

One has to show that wave-packet state vectors *do* exist after horizon exit. This will be the case if the phase of the mode function can be chosen so that it becomes real in the limit  $k/aH \rightarrow 0$ , so that [17,18]

$$\hat{\phi}_{\mathbf{k}}(\eta) \rightarrow v_{\mathbf{k}}(\eta) \hat{A}_{\mathbf{k}}, \quad (2)$$

where  $\hat{A}_{\mathbf{k}}$  is a constant operator (actually equal to  $\hat{a}_{\mathbf{k}} + \hat{a}_{-\mathbf{k}}^\dagger$ ). Indeed, in this limit a state vector which is an eigenvector of  $\hat{\phi}_{\mathbf{k}}$  at any instant remains an eigenvector.<sup>3</sup>

We need the condition (2) to be well-satisfied before the end of inflation, which we assume is sufficient for the perturbation to behave classically long after horizon exit. One might object that the fact the limit is never actually achieved makes this formalism non-rigorous, but the deep problems of interpretation which afflict quantum theory in a cosmological context mean that the criteria for exact classical behaviour are not accessible in our present state of knowledge. In any case, it seems unavoidable that (2) will be a necessary pre-requisite for classicality in any more complete theory that describes quantum processes in the Universe.

We will refer to fields satisfying Eq. (2) as light fields. This terminology is motivated by the example of a free scalar field with the canonical kinetic term and mass  $m$ , living in unperturbed de Sitter space corresponding to constant  $H$ . In that case the mode function becomes real if  $m^2 < (9/4)H^2$ . Assuming general relativity, the tensor metric perturbation has the dynam-

<sup>2</sup> An alternative is Warm Inflation, where one supposes that the cosmic fluid has a small thermalized component whose classical thermal fluctuation generates the curvature perturbation by freezing out at horizon exit [16]. Then the curvature perturbation is classical even before horizon exit, but the present discussion is still relevant for possible perturbations other than the curvature perturbation.

<sup>3</sup> Remember that all state vectors are time-independent in the Heisenberg picture. A derivation of classicality in the Schrödinger picture was given in [19,20] and its equivalence to the Heisenberg picture is discussed in [21].

ics of a massless canonically-normalized scalar field, and has little effect on the expansion so that it will certainly be a light field. A massless spin-1 field with minimal kinetic term is *not* light in our sense because its contribution to the action is invariant under a conformal transformation to flat spacetime, which means that its mode function continues to oscillate after horizon exit. Such a field will be light if it has a sufficiently small and slowly-varying mass (generated by a Higgs mechanism and/or a suitable non-minimal kinetic term), which nevertheless is big enough for Eq. (2) to be satisfied before the end of inflation. A gauge field of this sort might create a primordial magnetic field, or have more dramatic [22] effects. These are the only types of bosonic field that are usually considered in the context of quantum field theory, but if others exist our treatment will cover them.

The condition (2) is sometimes called WKB classicality. Once it is satisfied one might worry about the ‘Schrödinger’s Cat’ problem of how the initial state collapses into a particular state with definite  $\phi_{\mathbf{k}}$ . There is also the issue of decoherence, which addresses the question of why definite values of  $\phi_{\mathbf{k}}$  are preferred (why these are ‘pointer states’).

Our aim though, is to show that WKB classicality occurs also in the presence of interactions. In preparation for that, we write Eq. (2) in the equivalent form

$$[\hat{\phi}_{\mathbf{k}}, \hat{\phi}'_{\mathbf{k}}] \rightarrow 0. \quad (3)$$

Indeed, Eq. (2) obviously implies Eq. (3). Conversely, Eq. (3) implies that eigenvectors of  $\hat{\phi}_{\mathbf{k}}$  become also eigenvectors of  $\hat{\phi}'_{\mathbf{k}}$  because the eigenvalues of  $\hat{\phi}_{\mathbf{k}}$  are non-degenerate. Thus Eq. (3) implies Eq. (2), establishing their equivalence. We note that the field  $\hat{\phi}_{\mathbf{k}}$  which appears in Eq. (3) is chosen to be in the Heisenberg picture, in which state vectors are time-independent while the fields evolve. Therefore, this is the appropriate classicality condition for both free and interacting fields.

### 3. Higher order

At first order the only connected correlator is the two-point one, so that the perturbations are Gaussian. Non-Gaussianity will be generated at higher order, and may be a valuable discriminator between models. For this reason second order [2–5] and third order [6,7] calculations have been done, yielding estimates of respectively three-point and connected four-point correlators.

Higher order effects will come from ordinary interactions that exist even in flat spacetime, and from the gravitation theory which determines the metric perturbations. To handle them we can write  $\hat{H} = \hat{H}_0 + \hat{H}_I$ , and work in the interaction picture so that the evolution of the perturbations is given by  $\hat{\phi}'_{\mathbf{k}} = i[\hat{H}_0, \hat{\phi}_{\mathbf{k}}]$ . The term  $\hat{H}_I$  involves all of the fields, bosonic and fermionic, in the curved spacetime quantum field theory that is being invoked. From now on we drop the subscript  $\mathbf{k}$ .

An operator  $U$  transforms each operator  $A$  to the Heisenberg picture through  $A_{\text{hp}}(t) = U^{-1}(t)A(t)U(t)$ . The form of  $U$  is determined by  $A'_{\text{hp}} = i[H_{\text{hp}}, A_{\text{hp}}]$ , giving  $\dot{U}(t) = -i\hat{H}_I(t)U(t)$ , whose solution is

$$U(t) = T \exp \left( -i \int_{-\infty}^t \hat{H}_I(t') dt' \right) \quad (4)$$

where  $T$  is the time-ordering operator.

Eq. (4) has the formal power series expansion

$$U(t) = 1 + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \int dt' \cdots dt'' T[\hat{H}_I(t') \cdots \hat{H}_I(t'')], \quad (5)$$

which can be truncated at any required order  $n$  when  $\hat{H}_I$  is perturbatively small. The vacuum expectation values of products of the perturbations at a fixed time (correlators) can then be calculated to this order, using the Schwinger formalism, as described in [2,8,23,24] which is known as the ‘in-in’ or ‘closed time path’ formalism. Each correlator is given by a sum of Feynman diagrams [8].

From the discussion after Eq. (3), we see that WKB classicality requires  $[\hat{\phi}_{\text{hp}}, \hat{\phi}'_{\text{hp}}] \rightarrow 0$ . Written in terms of interaction picture quantities this requirement becomes  $UBU^{-1} \rightarrow 0$ , where

$$B \equiv i[\hat{\phi}, [\hat{H}_I, \hat{\phi}]] + [\hat{\phi}, \hat{\phi}']. \quad (6)$$

The second term of  $B$  tends to zero in accordance with Eq. (3). The first term is zero if  $H_I$  does not involve  $\hat{\phi}'$ , since  $\hat{\phi}$  commutes at equal times with every other interaction-picture field. If  $H_I$  does contain derivatives of the fields (which is the case in almost all interesting examples), then the first term will also approach zero as Eq. (3) becomes increasingly well-satisfied.

Since  $B$  is approaching zero, the WKB classicality requirement will therefore be satisfied if  $U \simeq 1$  to sufficient accuracy. This will be the case whenever perturbation theory applies, and could even be the case in a mildly non-perturbative situation. The requirement  $U \simeq 1$  is of course essential, since complete freedom to choose  $U$  would allow us to choose  $\hat{H}_I = \hat{H}_{\text{new}} - \hat{H}_0$  for any  $\hat{H}_{\text{new}}$ , leading to the wrong conclusion that all quantum fluctuations become classical after horizon exit. As we noticed earlier, a very massive scalar field fluctuation does not become classical, and neither does a massless vector field fluctuation. It is actually safest to include all mass terms in  $H_0$ , though that is not compulsory in the cosmological context because well before horizon exit they become negligible anyway.

### 4. Discussion

It is instructive to consider the implications of our result in a definite setting, which we take to be the one described in [2,4,8]. The action is Einstein–Hilbert with canonical kinetic terms for the light scalar fields. Around the time of horizon exit for the chosen scale, slow-roll inflation occurs in the direction of the inflaton  $\phi$ , and any orthogonal light fields  $\sigma_i$  have no effect at that time. The light scalar field perturbations may be defined on ‘flat’ spacetime slices of fixed  $t$ , defined as those whose metric is of the form  $g_{ij} = a^2(t)[\exp(\tilde{h})]_{ij}$ , with  $\tilde{h}(\mathbf{x}, t)$  transverse and traceless. Instead of the flat slicing one can choose a fixed- $t$  slice of uniform energy density, writing  $g_{ij} = a^2(t) \exp(2\zeta)[\exp(h)]_{ij}$ , with  $h(\mathbf{x}, t)$  transverse

and traceless. Then  $\delta\phi$  vanishes and the curvature perturbation  $\zeta(\mathbf{x}, t)$  becomes the degree of freedom.

Now consider an  $n$ -point correlator  $\langle\delta\phi_{\mathbf{k}_1}\cdots\delta\phi_{\mathbf{k}_n}\rangle$  of light fields. Once the condition (3) becomes well-satisfied, our result implies that such a correlation function can be computed by spatially averaging an effective classical field  $\delta\phi_{\mathbf{k}}^{\text{cl}}$ . It is now possible to make an immediate application to a calculation of Weinberg [25]. He finds, under stated assumptions but to all orders in perturbation theory, that no correlator of light field perturbations increases as fast as a power of the scale factor. Weinberg's argument can actually be applied directly to the effective classical field  $\delta\phi_{\mathbf{k}}^{\text{cl}}$ . To be precise, the quantity  $Q$  which appears in Eq. (7) of [25] can refer to the time dependence of  $\delta\phi^{\text{cl}}$  rather than merely its expectation value. Because the correlators are obtained by spatially averaging the effective field, there is no need to consider  $Q$ s formed from a product of perturbations.

Now we come to a more general consideration. Suppose that the values of some set of light scalar fields  $\phi_i(\mathbf{x})$ , evaluated at some epoch during inflation and smoothed on a scale bigger than  $H^{-1}$  so that they are classical, determine the future evolution of the locally-defined scale factor  $a(t)\exp[\zeta(\mathbf{x}, t)]$  at each position  $\mathbf{x}$ . Then [26,27]

$$\zeta(\mathbf{x}, t) = \sum_i N_i(t)\delta\phi_i(\mathbf{x}) + \sum_{ij} \frac{1}{2}N_{ij}(t)\delta\phi_i(\mathbf{x})\delta\phi_j(\mathbf{x}) + \cdots, \quad (7)$$

where  $N_i \equiv \partial N(\phi_i, \rho(t))/\partial\phi_i$ , etc., and  $N$  is the number of  $e$ -folds evaluated in a family of unperturbed universes, starting from an epoch when the light fields have assigned values and ending when the energy density has an assigned value.

This expression gives the correlators of  $\zeta$  in terms of the correlators of the light fields, which in turn are given by the in-in Feynman diagrams of quantum field theory. Let us call those Q-Feynman diagrams, since they originate from the quantum Schwinger formalism. But the evaluation of the correlators from Eq. (7) can itself be represented [9,27–29] by Feynman diagrams involving integrations over products of Fourier components of  $\delta\phi_i$ . These are in the classical regime by virtue of the smoothing, so we refer to them as C-Feynman diagrams. The curvature perturbation, then, can be obtained by combining Q-Feynman diagrams referring to the flat slicing, with the C-Feynman diagrams that take us from the flat slicing to the uniform-density slicing on which  $\zeta$  is defined.

On the other hand,  $\zeta$  during inflation can instead be calculated directly on the uniform-density slicing, which can be represented by its own set of Q-Feynman diagrams. We conclude that the latter set is equivalent to the combination of Q-Feynman and C-Feynman diagrams described in the previous paragraph. A specific example of this equivalence is provided by the tree-level calculation of the three-point correlator of  $\zeta$  in the single-field slow-roll inflation, which has been done on both the uniform-density slicing [2] and on the flat slicing [4] to obtain the same result.

Regarding the equivalence, it is important to realise that the epithet 'classical' refers *only* to the time evolution. Despite the epithet, each light field perturbation  $\delta\phi_i$  vanishes in the

limit  $\hbar \rightarrow 0$ , even after horizon exit, because it originates as a vacuum fluctuation. Therefore, our result is not inconsistent with well-understood effects such as quantum particle creation. However, it places non-trivial constraints on their subsequent evolution once  $k/aH \ll 1$ . The same is true for the C-Feynman diagrams, in which each loop introduces a factor  $\hbar$ , just like each loop of an Q-Feynman diagram. This is why the tree-level calculations mentioned earlier are equivalent; they both are of first order in an expansion in powers of  $\hbar$ . It remains to be seen how this type of argument goes for more general calculations, involving higher correlators and loop contributions, for which the evolution equation for the effective classical field  $\delta\phi^{\text{cl}}$  will need to be understood in greater detail.

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